RC SLAB DESIGN (BS8110:PART1:1997)

TWO WAY SPANNING SLAB DEFINITION - RESTRAINED

Overall depth of slab $h = 250$ mm

Outer sagging steel
- Cover to outer tension reinforcement resisting sagging $c_{saq} = 50$ mm
- Trial bar diameter $D_{tryx} = 25$ mm
- Depth to outer tension steel (resisting sagging) $d_x = h - c_{saq} - D_{tryx}/2 = 188$ mm

Inner sagging steel
- Trial bar diameter $D_{tryy} = 25$ mm
- Depth to inner tension steel (resisting sagging) $d_y = h - c_{saq} - D_{tryx} - D_{tryy}/2 = 163$ mm

Outer hogging steel
- Cover to outer tension reinforcement resisting hogging $c_{shog} = 50$ mm
- Trial bar diameter $D_{tryxhog} = 16$ mm
- Depth to outer tension steel (resisting hogging) $d_{xhog} = h - c_{shog} - D_{tryxhog}/2 = 192$ mm

Inner hogging steel
- Trial bar diameter $D_{tryyhog} = 16$ mm
- Depth to inner tension steel (resisting hogging) $d_{yhog} = h - c_{shog} - D_{tryyhog} - D_{tryyhog}/2 = 176$ mm

Materials
- Characteristic strength of reinforcement $f_y = 500$ N/mm²
- Characteristic strength of concrete $f_{cu} = 40$ N/mm²

![Slab Design Diagram](image)
RESTRAINED – 2 WAY SPANNING (CL 3.5.3)

MAXIMUM DESIGN MOMENTS

Length of shorter side of slab  \( l_x = 5.500 \text{ m} \)
Length of longer side of slab  \( l_y = 7.500 \text{ m} \)
Design ultimate load per unit area  \( n_s = 16.0 \text{ kN/m}^2 \)

Edge condition shorter side (1)  \( \text{Edge}_1 = "C" \)
Edge condition other shorter side (2)  \( \text{Edge}_2 = "C" \)
Edge condition longer side (3)  \( \text{Edge}_3 = "C" \)
Edge condition other longer side (4)  \( \text{Edge}_4 = "C" \)

Number of discontinuous edges  \( N_d = 0 \)

Moment coefficients

\[ \beta_{sy} = \frac{(24 + 2 \times N_d + 1.5 \times N_d^2)}{1000} = 0.024 \]
\[ \beta_1 = \text{if} (\text{Edge}_1 = "C", 4/3 \times \beta_{sy}, 0) = 0.032 \]
\[ \beta_2 = \text{if} (\text{Edge}_2 = "C", 4/3 \times \beta_{sy}, 0) = 0.032 \]
\[ \gamma = \frac{2}{9} \times \left[ 3 - \sqrt{(18) \times l_x l_y \times (\sqrt{\beta_{sy} + \beta_1} + \sqrt{\beta_{sy} + \beta_2})} \right] = 0.339 \]
\[ \beta_{3x} = \text{if} (\text{Edge}_3 = "C", 4/3, 1) = 1.333 \]
\[ \beta_{4x} = \text{if} (\text{Edge}_4 = "C", 4/3, 0) = 1.333 \]
\[ \beta_{sx} = \gamma / \left[ (1+\beta_{3x})^{0.5} + (1+\beta_{4x})^{0.5} \right] = 0.036 \]
\[ \beta_3 = \beta_{3x} \times \beta_{sx} = 0.048 \]
\[ \beta_4 = \beta_{4x} \times \beta_{sx} = 0.048 \]

Maximum span moments per unit width - restrained slabs

\[ m_{sx} = \beta_{sx} \times n_s \times l_x^2 = 17.6 \text{ kNm/m} \]
\[ m_{sy} = \beta_{sy} \times n_s \times l_y^2 = 11.6 \text{ kNm/m} \]

Maximum support moments per unit width - restrained slabs

\[ m_{sxhog} = \max(\beta_3, \beta_4) \times n_s \times l_x^2 = 23.5 \text{ kNm/m} \]
\[ m_{syhog} = \max(\beta_1, \beta_2) \times n_s \times l_y^2 = 15.5 \text{ kNm/m} \]

CONCRETE SLAB DESIGN – SAGGING – OUTER LAYER OF STEEL (CL 3.5.4)

Design sagging moment (per m width of slab)  \( m_{sx} = 17.6 \text{ kNm/m} \)
Moment Redistribution Factor  \( \beta_{sx} = 1.0 \)

Area of reinforcement required

\[ K_x = \text{abs}(m_{sx}) / (d_x^2 \times f_{cu}) = 0.013 \]
\[ K'_x = \min (0.156, (0.402 \times (\beta_{sx} - 0.4)) - (0.18 \times (\beta_{sx} - 0.4)^2)) = 0.156 \]

Outer compression steel not required to resist sagging

Slab requiring outer tension steel only - bars (sagging)
z_x = \min ((0.95 \times d_x),(d_x \times (0.5 + (0.25-K_x/0.9)))) = 178 \text{ mm}

Neutral axis depth \( x_x = (d_x - z_x) / 0.45 = 21 \text{ mm} \)

Area of tension steel required
\[ A_{sx,req} = \frac{abs(m_{sx})}{(1/\gamma_{ms} \times f_y \times z_x)} = 227 \text{ mm}^2/\text{m} \]

**Tension steel**

*Provide 16 dia bars @ 200 centres outer tension steel resisting sagging*
\[ A_{sx,prov} = A_{sx} = 1010 \text{ mm}^2/\text{m} \]

*Area of outer tension steel provided sufficient to resist sagging*

**Concrete Slab Design - Sagging - Inner layer of steel (cl. 3.5.4)**

Design sagging moment (per m width of slab) \( m_{sy} = 11.6 \text{ kNm/m} \)

Moment Redistribution Factor \( \beta_{sy} = 1.0 \)

Area of reinforcement required
\[ K_y = \frac{abs(m_{sy})}{(d_{y}^2 \times f_{cu})} = 0.011 \]
\[ K'_y = \min (0.156, (0.402 \times (\beta_{sy} - 0.4)) - (0.18 \times (\beta_{sy} - 0.4)^2)) = 0.156 \]

**Slab requiring inner tension steel only - bars (sagging)**
\[ z_y = \min ((0.95 \times d_y),(d_y \times (0.5 + (0.25-K_y/0.9)))) = 154 \text{ mm} \]

Neutral axis depth \( x_y = (d_y - z_y) / 0.45 = 18 \text{ mm} \)

Area of tension steel required
\[ A_{sy,req} = \frac{abs(m_{sy})}{(1/\gamma_{ms} \times f_y \times z_y)} = 173 \text{ mm}^2/\text{m} \]

**Tension steel**

*Provide 16 dia bars @ 200 centres inner tension steel resisting sagging*
\[ A_{sy,prov} = A_{sy} = 1010 \text{ mm}^2/\text{m} \]

*Area of inner tension steel provided sufficient to resist sagging*

**CONCRETE SLAB DESIGN – HOGGING – OUTER LAYER OF STEEL (CL 3.5.4)**

Design hogging moment (per m width of slab) \( m_{sxhog} = 23.5 \text{ kNm/m} \)

Moment Redistribution Factor \( \beta_{sx} = 1.0 \)

Area of reinforcement required
\[ K_{sxhog} = \frac{abs(m_{sxhog})}{(d_{sxhog}^2 \times f_{cu})} = 0.016 \]
\[ K'_{sx} = \min (0.156, (0.402 \times (\beta_{sx} - 0.4)) - (0.18 \times (\beta_{sx} - 0.4)^2)) = 0.156 \]

**Slab requiring outer tension steel only - bars (hogging)**
\[ z_{xhog} = \min ((0.95 \times d_{xhog}),(d_{xhog} \times (0.5 + (0.25-K_{xhog}/0.9)))) = 182 \text{ mm} \]

Neutral axis depth \( x_{xhog} = (d_{xhog} - z_{xhog}) / 0.45 = 21 \text{ mm} \)

Area of tension steel required
\[ A_{sxhog,req} = \frac{abs(m_{sxhog})}{(1/\gamma_{ms} \times f_y \times z_{xhog})} = 296 \text{ mm}^2/\text{m} \]

**Tension steel**

*Provide 16 dia bars @ 200 centres outer tension steel resisting hogging*
Concrete Slab Design - hogging - Inner layer of steel (cl. 3.5.4)

Design hogging moment (per m width of slab) \( m_{syhog} = 15.5 \text{ kNm/m} \)

Moment Redistribution Factor \( \beta_{by} = 1.0 \)

Area of reinforcement required

\[
K_{yhog} = \frac{|m|}{d_{yhog}^2 \times f_{cu}} = 0.013
\]

\[
K'_{y} = \min \left( 0.156, \frac{|m_{syhog}|}{d_{yhog}^2 \times f_{y} \times z_{yhog}} \right) = 0.156
\]

Slab requiring inner tension steel only - bars (hogging)

\[
z_{yhog} = \min \left( (0.95 \times d_{yhog}) \times (0.5 + \sqrt{0.25 - K_{yhog}/0.9}) \right) = 167 \text{ mm}
\]

Neutral axis depth \( x_{yhog} = \frac{d_{yhog} - z_{yhog}}{0.45} = 20 \text{ mm} \)

Area of tension steel required

\[
A_{syhog_{req}} = \frac{|m_{syhog}|}{1/\gamma_{ms} \times f_{y} \times z_{yhog}} = 213 \text{ mm}^2/\text{m}
\]

Tension steel

Provide 16 dia bars @ 200 centres Inner tension steel resisting hogging

Area of inner tension steel provided sufficient to resist hogging

Check min and max areas of steel resisting sagging

Total area of concrete \( A_c = h = 250000 \text{ mm}^2/\text{m} \)

Minimum % reinforcement \( k = 0.13 \% \)

\[
A_{st_{min}} = k \times A_c = 325 \text{ mm}^2/\text{m}
\]

\[
A_{st_{max}} = 4 \% \times A_c = 10000 \text{ mm}^2/\text{m}
\]

Steel defined:

Outer steel resisting sagging \( A_{sx_{prov}} = 1010 \text{ mm}^2/\text{m} \)

Area of outer steel provided (sagging) OK

Inner steel resisting sagging \( A_{sy_{prov}} = 1010 \text{ mm}^2/\text{m} \)

Area of inner steel provided (sagging) OK

Check min and max areas of steel resisting hogging

Total area of concrete \( A_c = h = 250000 \text{ mm}^2/\text{m} \)

Minimum % reinforcement \( k = 0.13 \% \)

\[
A_{st_{min}} = k \times A_c = 325 \text{ mm}^2/\text{m}
\]

\[
A_{st_{max}} = 4 \% \times A_c = 10000 \text{ mm}^2/\text{m}
\]

Steel defined:

Outer steel resisting hogging \( A_{sxhog_{prov}} = 1010 \text{ mm}^2/\text{m} \)

Area of outer steel provided (hogging) OK

Inner steel resisting hogging \( A_{syhog_{prov}} = 1010 \text{ mm}^2/\text{m} \)

Area of inner steel provided (hogging) OK
SHEAR RESISTANCE OF CONCRETE SLABS (CL 3.5.5)

Outer tension steel resisting sagging moments

- Depth to tension steel from compression face $d_x = 188$ mm
- Area of tension reinforcement provided (per m width of slab) $A_{sx,prov} = 1010$ mm$^2$/m
- Design ultimate shear force (per m width of slab) $V_x = 60$ kN/m
- Characteristic strength of concrete $f_{cu} = 40$ N/mm$^2$

Applied shear stress

$$v_x = \frac{V_x}{d_x} = 0.32 \text{ N/mm}^2$$

Check shear stress to clause 3.5.5.2

$$v_{allowable} = \min \left((0.8 \times \frac{1}{\sqrt{d}}) \times \sqrt{f_{cu}}, 5 \times \text{N/mm}^2\right) = 5.00 \text{ N/mm}^2$$

Shear stresses to clause 3.5.5.3

Design shear stress

$$f_{cu,\text{ratio}} = \frac{f_{cu}}{40 \text{ N/mm}^2} = 1.600$$

$$v_x = 0.79 \text{ N/mm}^2 \times \min(3,100 \times A_{sx,prov} / d_x)^{1/3} \times \max(0.67, (400 \text{ mm} / d_x)^{1/4}) / 1.25 \times f_{cu,\text{ratio}}^{1/3}$$

Applied shear stress

$$v_x = 0.32 \text{ N/mm}^2$$

SHEAR RESISTANCE OF CONCRETE SLABS (CL 3.5.5)

Inner tension steel resisting sagging moments

- Depth to tension steel from compression face $d_y = 163$ mm
- Area of tension reinforcement provided (per m width of slab) $A_{sy,prov} = 1010$ mm$^2$/m
- Design ultimate shear force (per m width of slab) $V_y = 60$ kN/m
- Characteristic strength of concrete $f_{cu} = 40$ N/mm$^2$

Applied shear stress

$$v_y = \frac{V_y}{d_y} = 0.37 \text{ N/mm}^2$$

Check shear stress to clause 3.5.5.2

$$v_{allowable} = \min \left((0.8 \times \frac{1}{\sqrt{d}}) \times \sqrt{f_{cu}}, 5 \times \text{N/mm}^2\right) = 5.00 \text{ N/mm}^2$$

Shear stresses to clause 3.5.5.3

Design shear stress

$$f_{cu,\text{ratio}} = \frac{f_{cu}}{40 \text{ N/mm}^2} = 1.600$$

$$v_y = 0.79 \text{ N/mm}^2 \times \min(3,100 \times A_{sy,prov} / d_y)^{1/3} \times \max(0.67, (400 \text{ mm} / d_y)^{1/4}) / 1.25 \times f_{cu,\text{ratio}}^{1/3}$$

Applied shear stress

$$v_y = 0.37 \text{ N/mm}^2$$

No shear reinforcement required
SHEAR PERIMETERS FOR A RECTANGULAR CONCENTRATED LOAD (CL 3.7.7)

Length of loaded rectangle \( l = 400 \text{ mm} \)

Width of loaded rectangle \( w = 200 \text{ mm} \)

Depth to tension steel \( d_x = 188 \text{ mm} \)

Dimension from edge of load to shear perimeter \( l_p = k_p \times d_x = 281 \text{ mm} \) where \( k_p = 1.50 \)

For punching shear cases not affected by free edges or holes:

Total length of inner perimeter at edge of loaded area \( u_{0\_gen} = 2 \times (l + w) = 1200 \text{ mm} \)

Total length of outer perimeter at \( l_p \) from loaded area \( u_{\gen} = 2 \times (l + w) + 8 \times l_p = 3450 \text{ mm} \)

PUNCHING SHEAR AT CONCENTRATED LOADS (CL 3.7.7)

Tension steel resisting sagging

Total length of inner perimeter at edge of loaded area \( u_0 = 1200 \text{ mm} \)

Total length of outer perimeter at dimension \( l_p \) from loaded area \( u = 3450 \text{ mm} \)

Depth to outer steel \( d_x = 188 \text{ mm} \)

Depth to inner steel \( d_y = 163 \text{ mm} \)

Average depth to "tension" steel \( d_{av} = (d_x + d_y)/2 = 175.0 \text{ mm} \)

Area of outer steel per m effective through the perimeter \( A_{sx\_prov} = 1010 \text{ mm}^2/m \)

Area of inner steel per m effective through the perimeter \( A_{sy\_prov} = 1010 \text{ mm}^2/m \)

Max shear effective across either perimeter under consideration \( V_p = 60 \text{ kN} \)

Characteristic strength of concrete \( f_{cu} = 40 \text{ N/mm}^2 \)

Check shear stress to clause 3.7.7.2

\[
\sigma_{\text{allowable}} = \min \left( (0.8 \text{ N}^{1/2}/\text{mm}) \times \sqrt{(f_{cu})}, 5 \text{ N/mm}^2 \right) = 5.000 \text{ N/mm}^2
\]

Shear stress - OK

No shear reinforcement required

CONCRETE SLAB DEFLECTION CHECK (CL 3.5.7)

Slab span length \( l_x = 5.500 \text{ m} \)

Design ultimate moment in shorter span per m width \( m_{ux} = 18 \text{ kNm/m} \)

Depth to outer tension steel \( d_x = 188 \text{ mm} \)

Tension steel

Area of outer tension reinforcement provided \( A_{sx\_prov} = 1010 \text{ mm}^2/m \)
Area of tension reinforcement required \( A_{sx,req} = 227 \text{ mm}^2/\text{m} \)

Moment Redistribution Factor \( \beta_{bx} = 1.00 \)

**Modification Factors**

Basic span / effective depth ratio (Table 3.9) \( \text{ratio}_{\text{span/depth}} = 20 \)

The modification factor for spans in excess of 10m (ref. cl 3.4.6.4) has not been included.

\[
f_s = 2 \times f_y \times A_{sx,req} / \left( 3 \times A_{sx,prov} \times \beta_{bx} \right) = 75.0 \text{ N/mm}^2
\]

\[
\text{factor}_{\text{tens}} = \min \left( 2, 0.55 + \left( 477 \text{ N/mm}^2 - f_s \right) / \left( 120 \times \left( 0.9 \text{ N/mm}^2 + m_{sx} / d_x^2 \right) \right) \right) = 2.000
\]

**Calculate Maximum Span**

This is a simplified approach and further attention should be given where special circumstances exist. Refer to clauses 3.4.6.4 and 3.4.6.7.

Maximum span \( l_{\text{max}} = \text{ratio}_{\text{span/depth}} \times \text{factor}_{\text{tens}} \times d_x = 7.50 \text{ m} \)

**Check the actual beam span**

Actual span/depth ratio \( l_x / d_x = 29.33 \)

Span depth limit \( \text{ratio}_{\text{span/depth}} \times \text{factor}_{\text{tens}} = 40.00 \)

Span/Depth ratio check satisfied

**CHECK OF NOMINAL COVER (SAGGING) – (BS8110:PT 1, TABLE 3.4)**

Slab thickness \( h = 250 \text{ mm} \)

Effective depth to bottom outer tension reinforcement \( d_x = 187.5 \text{ mm} \)

Diameter of tension reinforcement \( D_x = 16 \text{ mm} \)

Diameter of links \( L_{\text{diax}} = 10 \text{ mm} \)

Cover to outer tension reinforcement

\[ c_{\text{tenx}} = h - d_x - D_x / 2 = 54.5 \text{ mm} \]

Nominal cover to links steel

\[ c_{\text{nomx}} = c_{\text{tenx}} - L_{\text{diax}} = 44.5 \text{ mm} \]

Permissable minimum nominal cover to all reinforcement (Table 3.4)

\[ c_{\text{min}} = 25 \text{ mm} \]

Cover over steel resisting sagging OK

**CHECK OF NOMINAL COVER (HOGGING) – (BS8110:PT 1, TABLE 3.4)**

Slab thickness \( h = 250 \text{ mm} \)

Effective depth to bottom outer tension reinforcement \( d_{x,hog} = 192.0 \text{ mm} \)

Diameter of tension reinforcement \( D_{x,hog} = 16 \text{ mm} \)

Diameter of links \( L_{\text{diax,hog}} = 10 \text{ mm} \)

Cover to outer tension reinforcement

\[ c_{\text{tenx,hog}} = h - d_{x,hog} - D_{x,hog} / 2 = 50.0 \text{ mm} \]

Nominal cover to links steel

\[ c_{\text{nomx,hog}} = c_{\text{tenx,hog}} - L_{\text{diax,hog}} = 40.0 \text{ mm} \]

Permissable minimum nominal cover to all reinforcement (Table 3.4)
C_{min} = 25 \text{ mm}

Cover OK over steel resisting hogging